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THE VIBRATION OF STEEL STACKS

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STRUCTURAL DIVISION

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THE VIBRATIONS OF STEEL STACKS

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SYNOPSIS

Few of the many instances of excessive vibrations of self-supported steel stacks have been reported in technical literature. There are few data as to the magnitude of the forces produced by periodic vortex discharge or as to the damping properties of the stacks in absorbing the energy resulting from these periodic forces.

In the following article the authors analyze the data they have been able to accumulate and give tentative values for these unknowns which appear to be in the right general order.

INTRODUCTION

During the construction of the Moss Landing Steam Plant of the Pacific Gas and Electric Company, the comparatively severe vibrations described in a companion paper (1) occurred. This company had previously experienced similar troubles hereinafter described at their Oleum Steam Plant. To investigate the matter, the Pacific Gas and Electric Company initiated the research herein described.

Symbols and Notation

The following notation, arranged in order of their use in the following article, has been used.

Symbol	Term and Definition	Units
P	Unit Force	psf
C_D	Coefficient of Drag varying with the shape of the body and with the Reynolds Number	Pure Number
ρ	Mass density of Standard air—0.00238 Slugs	lbs. sec. ^{2}/ft.^{4}

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Symbol	Term and Definition	Units
V	Velocity	fps
d	Diameter of stack or cylinder	ft
ν	Kinematic viscosity	$ft.^2/sec.$
R	Reynolds Number. Relation of pressure to friction forces.	Pure Number
S	Strouhal Number	Pure Number
f	Frequency of Vortex Shedding or of Vibration	cps
C_L	Coefficient of Lateral Forces resulting from Vortex shedding.	Pure Number
δ	Logarithmic Decrement	Pure Number
k	Spring constant of a vibrating body	lbs/ft
m	Mass of a vibrating body	$lb\text{-}sec^2/ft^2$
δ_{st}	Deflection which would be produced by the disturbing force acting statically.	ft.
fm	Magnification factor: The amount, depending on the damping characteristics, the static deflection is multiplied to determine the maximum amplitude.	Pure Number
A	Amplitude. In a vibrating body the greatest distance from the mean position.	ft.
t	Thickness of Stack Shell	ins.
n	E steel/E gunite	Pure Number
g	Acceleration of gravity	ft/Sec^2

Source of Vibrations

When a fluid, such as air, flows past any obstacle, e.g., a cylindrical stack, certain forces develop as a result of changes in the velocity and/or the direction of the stream.

The force, or unit drag on the obstruction may be expressed by the formula,

$$P = C_D \rho V^2/2 = 0.0019\ C_D V^2 \qquad (1)$$

C_D varies with the shape of the obstruction and also depends on the Reynolds Number R.

$$\text{For air, } R = Vd/\nu = 6380Vd \tag{2}$$

The range of Reynolds numbers between 120,000 and 250,000 is termed "critical." Within this range the air flow changes from "laminar" to "turbulent." Within the range of Reynolds numbers of 10,000 to 120,000 C_D has a value of about 1.2. Through the critical range this value drops to about 0.30. Some experiments show this value to be constant to a value of R up to 500,000. Sherlock and Stalker (3)* found values of 0.30 for R = 270,000 and 0.40 for R = 450,000. The authors have found no data for the values (R = 3,000,000 to 10,000,000) connected with stack vibrations but have assumed the value of 0.35.

When the wind stream passes a cylindrical obstruction, alternating vortices are shed periodically, forming the Karman vortex trail. The pattern of the trail and the spacing of the vortices is shown by Figure 1. The frequency of these vortices is determined by the Strouhal Number S.

$$S = fd/V \approx 0.19 \tag{3}$$

Most experimentation as to the value of the Strouhal Number has been in the subcritical range (R less than 120,000). Relf and Simmons (4) found this number increased with larger Reynolds numbers. Other experimenters have reached the conclusion that with turbulent flow the shedding of the vortices became aperiodic.

As in the case of suspension bridges (5), the vibrations of stacks may be considered a case of self excited vibrations (negative air damping) rather than one of forced vibrations at resonance. The frequency of vortex shedding is determined by the natural frequency of the stack rather than by the velocity of the approaching air. The only significance of the Strouhal number seems to be that it indicates the velocity at which the vortex shedding, and therefore the response of the stack, will be most vigorous.

In the experimental work on suspension bridges (5) two types of self excited vibrations were observed. In the first "restricted" type, vibrations in a given mode occur over a considerable range of velocity reaching a maximum amplitude at a certain wind velocity. As the velocity is increased above this point, the amplitudes decrease. When the velocities are still further increased, vibrations in the same mode reoccur, at a velocity twice that of the first appearance. In the second "catastrophic" type, vibrations start at some critical wind velocity and, as the velocity increases, the amplitudes build up rapidly resulting in the failure of the structure.

The writers have assumed, possibly without sufficient evidence, that the vibrations of stacks were of the restricted type and that if the stacks were so designed that no damaging stresses would occur during the occurrence of the lower critical velocity, such stacks would be safe for all wind velocities below twice the lower critical.

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*Numbers in text within parentheses refer to the bibliography.

Lateral Forces

As Karman vortices are alternately discharged, the velocities and therefore, by Bernoulli's theorem, the pressures on opposite sides of the cylinder vary, producing an alternating force,

$$P_L = C_L \rho V^2/2 \qquad (4)$$

The writers have found no data as to the values of C_L within the range of Reynolds Numbers herein considered. For lower Reynolds Numbers, Steinman (8) gives values of $1.9C_D$. In the absence of more complete information for high Reynolds Numbers, the authors have assumed $C_D = 0.35$ and $C_L = 0.66$.

The amplitudes of vibration which will result from these alternating periodic forces depends on (a) the magnitude of the alternating forces, or in lieu thereof, the energy input per cycle, (b) the number of cycles which are applied and (c) the damping characteristics. Of these, the assumptions as to the alternating forces have been given above. At critical velocities the maximum amplitudes build up rapidly until the "steady state" at which the input and dissipation of energy per cycle are equal.

Damping

Damping forces are those which absorb energy from any vibrating body. In the case of stacks, these forces are the sum of (a) the structural damping resulting from the hysteresis effect in stress reversal; (b) the energy dissipated at the foundations; (c) the aerodynamic damping or friction between the air and the vibrating stacks; and (d) the energy dissipated by any external energy absorbers that may be used.

In an unlined welded stack, the only structural damping within the stack itself arises from the hysteresis effect. In the stress-strain diagram, most of the elongation is elastic. This is accompanied by a small amount of plastic deformation. During each vibration cycle the energy, ΔW, within the hysteresis loop is dissipated by heat. The damping ratio Ψ, equals $\Delta W/W$, W being the work of deformation stored in the extreme position.

The damping ratio may also be expressed by the logarithmic decrement, δ. The successive amplitudes may be observed after the disturbing force has been removed. If the envelope of the successive amplitudes are plotted, δ may be calculated. (11)* For small damping, $\delta = \Psi/2$.

There are only limited data as to the damping ratios of various structural materials. It may be noted that most of the excessive stack vibrations have occurred in welded stacks and confirms the experimental results that damping is greatly increased by the friction on faying surfaces of riveted construction. Since the ratio of plastic to total elongation increased with the unit stress, δ is not constant but increases with the stress.

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*See Figure 23 of this reference.

Teller and Wiles (9) tested bolted trusses and solid sections. At unit stresses of 2 ksi the respective decrements were 0.009 and 0.003 at 10 ksi were 0.048 and 0.007.

Since the stress-strain curve for concrete does not follow Hooke's Law as closely as in the case of steel, it may be inferred that the damping characteristics thereof will be considerably greater. Muckenhoupt (10) found values of 0.10 for δ at stresses of 200 psi with rapid increases as the ultimate tensile strength was approached.

In many investigations of vibrations, it has been found that the greatest source of damping was in the energy absorbed by the foundations. Again, few specific data applicable to stacks are available.

With the comparatively small velocities that are involved, it is probable that positive aerodynamic damping in still air, or with a wind of constant velocity, is negligible.

Vibration Frequencies

The general formula for the natural frequency of a vibrating system is:

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \tag{5}$$

With damping (assumed viscous and of small amount) the amplitude may be determined by

$$A = \delta_{st} \cdot f_m = \pi \delta_{st} / \delta \tag{6}$$

The significance of the "magnification factor" becomes apparent with δ for steel being only about 0.007 and $f_m \approx 400$.

In investigating the probabilities of resonance with the alternating forces, a calculation of the frequency of the stack being designed is necessary. For a cantilever of uniform section, fixed at its base, Eq. (5) reduces to:

$$f = \frac{C}{2\pi l^2} \sqrt{\frac{EIg}{w}} \tag{7}$$

in which C is a constant depending on the mode of vibration.

C_1 (fundamental) = 3.515   l = length of cantilever in feet

C_2 = 22.0   w = weight in pounds per foot

C_3 = 61.7

Generally the sections of stacks are not uniform: the plate thickness, weight, and often the diameter decreasing from the bottom to the top. In such cases, the frequency may be found by the Rayleigh-Ritz method of summation.

$$f = \frac{1}{2\pi} \sqrt{\frac{gWx}{\Sigma Wx^2}} \tag{8}$$

In the solution of this equation, it is necessary to divide the height of the stack into a number of sections, say twenty. W is the weight of each section and x is the resulting dead load deflection at the center of each section for the stack acting as a horizontal cantilever beam.

Preliminary estimates of the frequency of an unlined stack may be made from the formula:

$$f = d\,(3040t+2520)\,/\,h^2 \qquad \text{(8a)}$$

in which

t = thickness of shell, inches, at bottom of vertical section of stack

h = equivalent height for vibration calculation = $h_c + h_f/3 + d/2$

h_c = height of straight shaft

h_f = height of flare

d = diameter of straight portion, feet

A comparison of calculated and observed frequencies for the examples herein reported follows:

	Frequency, Cycles per second			
	Calculated			
	Eq.(8)	Eq.(8a)	Observed	Calc/Observed
Steel tube, plain	3.94*	3.68	3.75	1.05
Steel tube, gunite encasement	2.72**	xxx	2.20	1.24
Moss Landing, unlined	1.23	0.63	1.12	1.10
Moss Landing, lined	0.85	xxx	0.82	1.04
Contra Costa, unlined	1.12	1.06	0.97	1.16
Contra Costa, lined	0.75	xxx	0.71	1.05
Michigan, unlined	1.24	1.24	1.00	1.24

*Eq (7)

**$n = E_g/E_s = 20$

xxx Eq. 8a inapplicable to lined stacks

From the above it will be seen that the observed frequency is less than that calculated. It is probable that the stacks are not completely fixed at their base and that there is some movement of the foundations. It appears a reasonable assumption to divide the calculated frequency by 1.15 for unlined stacks. For gunite lined stacks using n = 20, the corresponding factor becomes about 1.05.

To date the authors have found no data as to the interaction between steel and a brick lining.

In some cases, the variations are such that they may be approximated by sections with such variables that Eq. (8) may be integrated. Some of these cases are given by Timoshenko (12).

The frequency of ovalling, or breathing of an unlined stack (11) in the first mode is given by:

$$f = 24500\ t/r^2 \quad \text{(All dimensions in inches)} \qquad (9)$$

Experiments and Observations

Because of its importance in the design of stacks, the Pacific Gas and Electric Company authorized a test investigation as to the best means of restricting vibrations in stacks to amplitudes that would not involve serious stresses. This investigation has covered, mainly, the damping effects of a gunite lining.

A. Model Tests

Preliminary tests were made in the laboratory on a model consisting of a 6 inch steel pipe cantilevered from a concrete base. Vibrations were initiated by suddenly releasing a horizontal pull applied at the free end. The essential results are given by Table I on which the following comments may be made:

1. The logarithmic decrement of .007 for the unlined stack is in line with those given above. With the smaller stresses as the amplitudes decreased, there was some decrease in the decrement.

2. With the gunite encased specimen, the decrement was increased nearly 30 times. In this case, however, the decrement increased with decreasing amplitudes (characteristic of Coulomb friction damping (6)).

3. At the maximum amplitudes the stress in the concrete at the base would be approximately 320 psi, probably approaching its ultimate strength. The frequency calculations based on the value of EI determined by the dynamometer test check well with the observations. Evidently for these conditions the analysis should be made on the basis of a "cracked" section or the value of n increased. For gunite with lightweight aggregate, a value of n = 20 appears reasonable.

4. The values of both the frequency and the decrement indicate a truly fixed condition at the base.

B. Moss Landing Stacks

The vibrations observed at the Moss Landing stacks prior to their lining, the tests made on the stacks, and the measures employed to inhibit troubles in the completed stacks are described in a companion paper (1). The data have been made available to the authors of this paper. The conclusions drawn herein may not agree with those of the other paper.

(a) Breathing

Substituting the value of 68 inches for r in formula (9), the theoretical frequency for the 5/16" plates would be 1.67 cps or 100 cpm, a somewhat higher value than the 88 cpm as observed.

Assuming the latter value and also the estimated wind velocity of 25 mph (37 fps) as correct the Strouhal number would become 0.45. R would be 2,650,000.

In discussing the above there are several possibilities:

1. The velocity of 25 mph was estimated only. Again assuming the estimate correct near the ground, the velocity (Ref. 15, Fig 3) at a height of 200 ft. would be approximately 48 fps. With a frequency of 1.47 cps, the Strouhal Number would become 0.35.
2. Assuming a Strouhal number of 0.19, the critical velocity (Eq. 3) would become 88 fps or about double that assumed above. It is possible that the frequency of vibration was twice that of the vortex shedding.
3. When the breathing was observed there were only inappreciable movements of the stacks as cantilevers so that the stacks may be considered as fixed in space. We believe the evidence conclusive that with cantilever vibrations the value of S is constant within the range of Reynolds numbers herein considered. The higher values of S as found by Relf and Simmons (4) may apply for fixed cylinders.

(b) Cantilever Vibrations

In the cantilever vibrations of October 18, 1949, the wind velocity was estimated at 30-40 mph and the frequency observed at 73 cpm. In the tests to determine the damping properties of the stack, the frequency was 67 cpm. Assuming a frequency of 1.20 cps and a Strouhal number of 0.19 the wind velocity would be 72 fps or 49 mph. Perhaps the velocity at the top of the stack was in this general order.

Assuming a wind velocity of 72 fps, and C_L at 0.66, p_L from Eq. (4) would be 4.1 psf of stack. For such load applied over the cylindrical section of the stack only, the static deflection at the top of the stack has been calculated at 0.88". The maximum observed amplitude was 16 inches resulting in a magnification factor of about 18.

A. Contra Costa Stacks

The Contra Costa Steam Plant, when completed, will have a total capacity pf 570,000 kw. There are two sets of three stacks, one for each boiler, the axis of each set being due north. Each stack is 11' -0" I.D. Starting from the top, the plates are 105' of 5/16" , 40' of 3/8" and 55' of 1/2" . All stacks were lined with 3" of light weight gunite, except that in the lower 51 ft. of the stack, the thickness tapered from 3" to 8" . The stacks were designed for a static wind pressure of 20 psf of projected area.

The properties of the gunite as determined by nine test cylinders at 28 days were:

	Range	Average
Compressive Strength, psi	2820 - 4300	3520
Unit Weight pcf	107.3 - 113.5	110.2
E pounds/inch ^{2}x10^{-6}	2.01 - 3.27	2.37

Vibration was initiated by releasing a horizontal force at the top of the stacks. Tables II and III give the ordinates of the decay curves for the unlined and the lined stacks respectively, and also the calculations for the logarithmic decrement.

The observed vs the calculated frequencies have been tabulated above. A plotting of decrements vs amplitudes indicates a very considerable increase of decrement with the larger amplitudes or stresses.

Because of the experience at Moss Landing, the Contra Costa stacks were guyed immediately upon their erection, the guys being removed upon the completion of the lining.

With a Strouhal number of 0.19, and a frequency of 42.5 cpm (0.71 cps), the critical velocity of the lined stacks would be 42.6 fps or 29 mph. With this velocity, the stagnation pressure would be 2.14 pounds per square foot. Using C_L of 0.66, the lateral force would be 16.0 pounds per lineal foot of stack. The static deflection at the top of the stack under this force would be 0.30 inches.

Since the completion of the lining, several wind velocities of 30 mph have occurred. The maximum observed amplitude has been 3 inches, corresponding to a magnification factor of 10. With this magnification factor, the equivalent wind pressure would be 21.4 psf, and the unit stress at the base of the stack, based on composite action, 3500 psi.

The question arises as to what would happen in the case (not anticipated in this particular location) of a wind twice the critical. If vortices of four times the intensity were shed, the deflections at the top of the stack would be about 12 inches and the unit stress in the shell 14000 psi.

B. Oleum Steam Plant

At the Oleum, California Steam Plant of the Pacific Gas and Electric Company, there are three stacks at 31 feet centers. The stacks are 200 feet high. The upper 110 feet have an outside diameter of 6' -6"; the lower part tapers to a diameter of 9 feet at the base. In the original design, there was a heavy truss 10 feet deep, 100 feet above the base tying the stacks together. The design also provided for gunite lining.

On August 11, 1941, before the gunite lining had been applied, two or more or the stacks vibrated violently in the wind. A crack developed in the shell of the center stack at a point just above the truss. The wind velocity was estimated at 45 mph, and in a direction nearly parallel to the axis of the group. An eye witness states that the tops of the stacks were moving in opposite directions and normal to the direction of the wind. No estimate of the frequency is available. It may or may not be pertinent that the spacing of the stacks is close to the spacing, 4.31d, of the Karman vortices. Without the truss at Elev. 100, the frequency of the stacks would have been about 1.0 cps, and the critical velocity (assuming S - 0.19) about 20 mph. Without definite knowledge as to the

direction of the wind, it becomes impossible to calculate the effect of the truss on the period. It is possible that the vibrations resulted from a wind twice the critical.

As a corrective measure, a horizontally rigid truss was installed near the top of the stacks and the stacks were lined with gunite. No further vibrations with material amplitudes have been observed since.

C. Baltimore Stacks

The vibrations of the two stacks at the Gould Street station of the Consolidated Gas, Electric Light and Power Company of Baltimore have been described by Pagon (2). Observations pertinent to this investigation are:

(1) The critical velocity was 53 mph or 78 fps with a frequency of 1.33 cps. With a stack diameter of 11 feet, the Strouhal number was 0.187 at a Reynolds number of 5,500,000.

(2) Vibrations were observed in the leeward stack only. The stacks are spaced 48 feet on centers or 4.36 d as compared to 4.31 d for the spacing of the Karman vortices.

(3) Very severe ovalling occurred with a frequency of 2.20 cps. By Eq. (9), the frequency would be 1.75 and 1.40 cps for the 5/16 and 1/4 inch plates respectively. Pagon's calculations (2) give corresponding frequencies of 2.30 and 1.90 respectively.

D. Excerpt from O.N.I. Report - 99

Description of Typhoon of August 3, 1941 on Guam

"Maximum wind velocity 125 miles per hour

............We also watched the Power Plant stack, which performed again its interesting "typhoon shimmy." This stack, 9 feet in diameter and 150 feet high, reacts to heavy winds in a manner which has to be seen to be believed. Its movement is quite as spectacular, in fact, as was that of the Tacoma Narrows Bridge before its collapse, which some of us have seen in the movies. The stack movement consists of a rhythmic panting, with the cross section becoming oval first in one direction and then in the other, so that at one instant the upper section appears to be perhaps eleven feet in diameter, the next instant only seven. The motion is very rapid — we timed it at 105 vibrations per minute —and, when seen from below, is accompanied also by a hula twist, difficult to describe. But on the question of the safety of the stack we had no qualms. It has weathered the typhoon of November 3rd with only minor damage, had recently been shortened, examined, and repaired, and so, in spite of these gyrations, we expected it to last. It did."

The authors have no details of this stack. Assuming shell thickness at the top of 1/4 inch and at the bottom of 1/2 inch, the frequency as a cantilever from Eq. (8a) would be 1.86 cps or 112 cpm, and the frequency of the 1/4" plate in breathing would be 2.1 cps or 125 cpm both being fairly close to the observed 105 cpm. The Strouhal number would be 0.105.

The observations indicate, and the calculations confirm the simultaneous occurrence of breathing and cantilever vibrations. There is no record of a Strouhal number as low as that given above. The most probable explanation seems that of a critical velocity of the stacks as a cantilever at about 60 mph and of the stack responding either to vortices shed at twice the frequency of the stack or of the stack controlling the vortex shedding with the vortices being shed at half the usual rate.

With a velocity of 120 mph the stagnation pressure would be 37 psf. With a C_L of 0.66 the lateral pressure would be about 25 psf. Without details of the design, it would appear that, in this case, the magnification factor was comparatively low.

E. Pittsburgh Stacks

On June 12, 1951, observations were recorded of the vibrations of stacks near Pittsburgh, Pa. 250' high with an 8' -7" dia. at the top. Shell thicknesses were 95 ft. of 5/16", 40 ft. of 3/8", 40 ft. of 7/16", and 75 ft. of 1/2" plate. The vertical section was 200 ft., flaring to a diameter of 11' -8" at the base. Deflections of 24" to 30" on either side of the center line were noted with the wind blowing at 17 mph. These deflections were transverse to the direction of the wind indicating that they were the result of aerodynamic forces synchronized with the natural mode of vibration of the stacks.

The frequency of these stacks calculated from Eq. 8 is 0.635 cps. Using a factor of 1.15 to allow for the action at the base, the frequency becomes 0.55 cps. With a Strouhal number of 0.19, the critical velocity becomes 24.8 fps or 17 mph. With this velocity and C_L of 0.66 the lateral force would be 0.48 psf. The static deflection for such a load would be 0.45". With an observed amplitude of 30 inches, the magnification factor becomes 67.

Mr. E. Durand, Manager of Contract Engineering, Pittsburgh-Des Moines Company, has furnished the following:

"We have had several experiences with stacks vibrating at their natural period, such vibration being excited by resonant vortex shedding. We had serious trouble with a stack 6 ft. diameter x 179 ft. in height. The height above the bell in this case was 126 ft. 11-1/2 in., H/D ratio 21.2.

"We had moderate trouble with a stack 6 ft. 9 in. diameter x 150 ft. high in which the height above the bell was 100 ft., H/D 14.8.

"We had very serious trouble with several stacks which were 8 ft. 7 in. diameter x 250 ft. in height in which the height above the bell was 200 ft., the H/D 23.3.

"We have not found any way to effectively damp the vibrations where resonance occurs. Our present policy is to avoid stacks with a diameter to height ratio greater than 20."

F. Detroit, Michigan

Recently three stacks, 300 ft. high, were being erected near Detroit, Michigan. The stacks had a diameter of 16 ft. at the top, flaring to 18 ft. at elevation 96 ft. above the base and to 27 ft. at the base. From the

top shell thicknesses were 100 ft. of 1/4", 22 ft. of 5/16", 30 ft. of 3/8", 22 ft. of 7/16", 30 ft. of 1/2", 28 ft. of 9/16" and 68 ft. of 5/8".

On November 26, 1952, with two of the stacks completed except for the proposed gunite lining, and the third stack completed to within 40 ft. of the top, a severe storm occurred. At a nearby airport steady winds of 65 mph were reported with somewhat higher gust velocities. The two completed stacks swayed violently; the movements of the incompleted stack were negligible. The motion at the top of the completed stacks is reported to have been circular.

The two completed stacks failed at a point about 90 ft. from the top. Initial failure was by buckling followed by a tearing of the plate. After failure, the movement of the stacks decreased and the upper sections remained in place. There were also considerable movements at the base of the stacks. The concrete surrounding the base plate was cracked loose and some elongation of the anchor bolts occurred.

The hypotheses above presented may be applied to the above information.

Calculated frequency of stacks Eq. 8	1.24 cps
Correction factor.	1.15
Corrected frequency of stacks	1.08 cps
Observed frequency of stacks.	1.00 cps

Assuming $f = 1.08$
$S = 0.19$
$d = 16.5$ (Average for top 102′ of stack)

Critical velocity	94 fps = 64 mph
Stagnation Pressure	10.5 psf
C_L	0.66
Lateral pressure — static.	6.94 psf
Magnification factor (Assumed)	14.5
Equivalent Static Pressure	100.0 psf

Shell Thickness	Maximum Compression	Critical Buckling
Inches	Ksi.	Ksi.
0.25	12.0	12.4
0.312	14.0	15.2
0.375	17.4	17.9
0.438	19.4	20.5
0.500	22.6	23.1
0.562	19.6	22.5
0.625	16.8	19.3

Stress in Anchor Bolts: 54.5 ksi.

$R = 6380 \times 94 \times 16.5 = 9{,}900{,}000$

Discussing the above:

1. The magnification factor of 14.5 was assumed arbitrarily to produce results comparable with the buckling strength of the shell. It is somewhat lower than would have been assumed from other

data. Quite likely the taper of the stack had a beneficial effect in that the critical velocities, and therefore the maximum forces, would vary over the length of the stack.

2. That no appreciable vibrations occured in the incompleted stack is readily explained by the fact that the natural frequency of the shorter stacks was sufficiently greater that the critical wind velocity was not reached.

G. Vibrations of Concrete Stacks

While this paper is primarily directed to steel stacks, vibration of concrete stacks have been reported by Frank (15). Table I of his article follows:

Smoke Stack No.	Height Meters	Diameters Top	Bottom Meters	Average	Frequency cps	Critical mps	Velocity mph
1	167.52	8.40	13.00	10.70	0.39	28.0	62.7
2	110.00	5.36	8.15	6.76	0.40	18.0	40.3
3	100.23	5.50	8.23	6.87	0.435	20.1	45.0
4	100.00	5.62	11.80	8.71	0.585	34.1	76.2

The observed amplitude of stack No. 1 was 393 mm (1.29 ft.). This may indicate that at least considerable taper is necessary to avoid vibration from periodic wind forces.

Frank attributes the apparent circular orbit observed at the top of stacks to a combination of the lateral forces produced by the Karman vortices and longitudinal forces produced by gusts. He further advises that the frequency of the stack be kept as low as possible.

Discussion of Data

While the above data are incomplete, it is believed certain tentative conclusions may be drawn therefrom. It is hoped that the paper may arouse discussion which will improve them.

1. The damping of the unlined stacks at low amplitudes at both Contra Costa and Moss Landing was about eight times that which might have been anticipated from the damping characteristics of the welded steel. With higher amplitudes the damping increased. The most apparent conclusion is that the major source of energy dissipation was at the foundation. In both cases, the stack foundations consisted of a concrete mat supported by piles. Further observations are desirable to determine what decrements can safely be assumed for an unlined stack.

2. A greater amount of additional damping was expected from the gunite linings. However, the additional damping was considerable in view of the fact that, probably the damping of the foundation was about the same in both cases so that there was a material increase in the damping of the stack proper.

Other instances of stacks exhibiting excessive vibrations before lining and no appreciable vibrations thereafter are known to the authors. In one case, a brick lining was used. Possibly more friction would develop in this case than when a gunite lining is used.

3. A lining has the advantage of lowering the critical velocity provided it is not lowered to the point where the stack would be subject to subharmonic vibrations from winds of higher velocity.

4. In many cases of reported vibrations where there are two or more stacks in a group, it appears usual that large amplitudes occur on only one of the stacks. This is a matter on which the authors have been unable to find any satisfactory answer. The importance of lining may be emphasized by stating that no instance of excessive vibrations of lined stacks has become known to the authors.

Design of Stacks

The foregoing appears to permit some tentative suggestions for the design of stacks, insofar as vibrations are concerned.

1. The highest probable wind velocity should be assumed. Eliminating the tornado from consideration, velocities of 120 mph may occur in Florida hurricanes and slightly lower velocities along the Atlantic and Gulf Coast. Velocities of the same order may be anticipated on the summit of high hills. With these exceptions, a velocity of 80 mph may be considered a safe assumption for other locations within the continental United States.

2. It is possible to make the diameter of the stack sufficiently great that its natural frequency would be above that of the vortices. For stacks taller than 150 ft. this would usually result in a greater diameter of stack than is economical.

3. The stack might be designed as a frustum of a cone for its entire height. By this means the critical velocity would differ throughout its height. The moment of inertia of this type of section would be large in comparison with a similar stack of uniform section with a resulting large increase in the natural frequency of the stack vibration.

4. Where it is possible to introduce rigid bracing between the stack and adjoining structures, the frequency of the stacks may be increased by such bracing. The optimum location is at a point about 2/3 of the height of the stacks.

5. Where there are two or more adjoining stacks the vibrations may be inhibited by introducing a system of bracing near their tops. The probability of two or more stacks vibrating together as a result of vortex shedding appears remote.

6. Energy absorbing devices may be installed or the design may take advantage of the damping properties of masonry and concrete, which are much greater than those of steel.

7. The authors believe that generally the most economical procedure is to design the stacks so that their natural frequency is that of the vortex shedding at a wind velocity of 45 mph. With proper design the energy of the vortices at this velocity will not be sufficient to produce excessive stresses: the stack would be safe against excessive vibrations until a velocity of 80 mph is reached. It should be noted that this conclusion is based on the unconfirmed assumption of the restricted type of self induced vibrations.

Further Investigations

The design of stacks is a matter of considerable economic importance to power and other industrial companies. Further information is desirable to eliminate the uncertainties in many of the assumptions made by the authors. Among these uncertainties are:

1) The drag coefficient for cylinders with Reynolds Numbers in the range of 3 to 10 millions.

2) The drag coefficient for vibrating cylinders.

3) The corresponding lateral coefficients of force.

4) Confirmation or negation of the assumption as to the type of self-excited vibrations.

5) Damping characteristics for welded stacks, both lined and unlined and with varying types of foundation restraint. Considerable statistical data could be obtained from observations on existing stacks.

6) Design of any practical devices by which the shedding of periodic vortices may be eliminated.

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as Chief Power Engineer (Now Vice-President), W. L. Dickey as Chief Structural Engineer and G. B. Woodruff as Consultant. Mr. I. E. Boberg, Chief Engineer of the Chicago Bridge and Iron Company furnished the data for the Michigan stacks. The authors have borrowed freely from the material listed in the Bibliography.

BIBLIOGRAPHY

1. Dockstader, E. A., The Vibrations of Steel Stacks at the Moss Landing Plant of the Pacific Gas and Electric Company. A.S.C.E. Separate No. 511, September, 1954.

2. Pagon, W. Watters, Wind Design by Aerodynamic Principles. A series of eight articles in Engineering News-Record, March 15, 1934-October 31, 1935.

3. Sherlock, R. H. and Stalker, A Study of Flow Phenomena in the Wake of Smoke-stacks. Engineering Research Bulletin, No. 29, University of Michigan.

4. Reif, E. F. and Simmons, L. G. F., The Frequency of the Eddies Generated by the Motion of Circular Cylinders Through a Fluid — Reports and Memoranda No. 917, June, 1924, Technical Report of Aeronautical Research Committee for 1924-25 (London.)

5. Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge, University of Washington Engineering Experiment Station No. 116.

6. Aerodynamic Stability of Suspension Bridges, Progress Report of the Advisory Board on the Investigation of Suspension Bridges - A.S.C.E. Separate No. 144, August, 1952.

7. Sherlock, P. H. and Stout, M. B., Picturing the Structure of the Wind, Civil Engineering, June, 1932.

8. Steinman, D. B., Problems of Aerodynamic and Hydrodynamic Stability, Bulletin 31, University of Iowa Studies in Engineering, 1947.

9. Teller, L. W. and Wiles, E. G., Tests of Structural Damping Public Roads, October 1953.

10. Muckenhoupt, Carl F., Study of Concrete as a Material for Turbine Foundations.

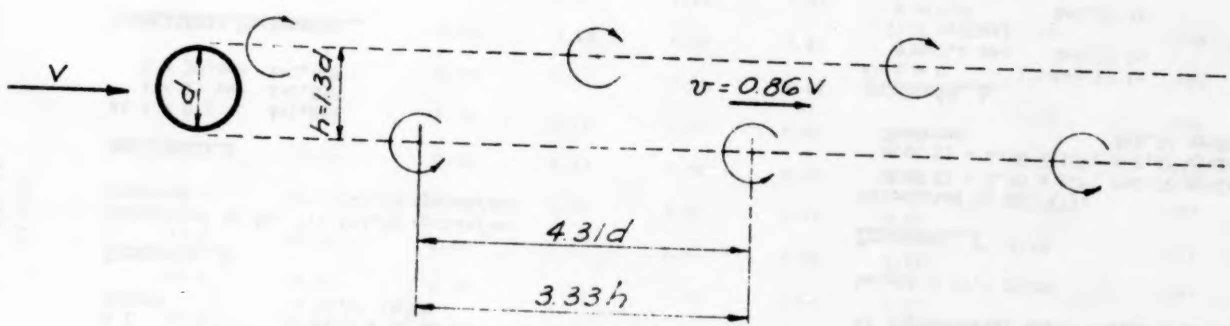
11. Timoshenko, S., Vibration Problems in Engineering.

12. Den Hartog, J. P., Mechanical Vibrations.

13. Sherlock, R. H., Variation of Wind Velocities and Gusts with Height, A.S.C.E. Separate No. 126, April 1952.

14. Proposed Standard Specifications for the Design and Construction of Reinforced Concrete Chimneys. A.C.I. Committee 505, Journal American Concrete Institute January, 1953.

15. Stromunserregte Schornsteinschingungen (Vibrations of Smokestacks Induced by Flow,) by Dr. Ing. Walther Frank, Prague, Volume 22, Issue 2 of the Journal Archiv fur Warmewirtschaft und Dampfkesselwesen, dated February, 1941.



KÁRMÁN VORTEX TRAIL IN WAKE OF CYLINDER

Fig. 1

TABLE I

VIBRATION TESTS ON STEEL TUBE

Plain Tube

Outside Diameter = 6.000 in.
Inside Diameter = 5.624 in.

Wall Thickness = 0.188 in.
Length = 20.15 ft.
$I = 14.42\ in^4$ = $0.100\ in^2\text{-}ft^2$
E (assumed) = $2.90 \times 10^7\ lb/in^2$
E I = $2.90 \times 10^6\ lb\text{-}ft^2$
Weight = 11.67 lb/ft

Frequency, f

Calculated by Eq. (7) f=3.94 cycles/sec
Observed f=3.75 cycles/sec

Amplitude, A

At t = 0 A=1.25"
t = 15 sec A=0.83"
t = 30 sec A=0.57"

Logarithmic Decrement,

t_0 to t_{15} δ =.0073
t_{15} to t_{30} δ =.0068
t_0 to t_{30} δ =.0070

Encased with 1.92" of Gunite

Outside Diameter = 9.84"
I_g = $396\ in^4$
n = 20 (assumed)
$E_g = 1.45 \times 10^6\ lb/in^2$
$E_g I_g / E_s = 19.8\ in^4$
$I_s + E_g I_g / E_s = 34.22\ in^4 = 0.238\ in^2\text{-}ft^2$
EI (calculated) = $6.90 \times 10^6\ lb\text{-}ft^2$
EI (dynamometer test) = $4.36 \times 10^6\ lb\text{-}ft^2$

Weight = 57.5 lb/ft

Frequency, f

Calculated by Eq. (7);
With EI = 6.90×10^6, f=2.72 cycles/sec
With EI = 4.36×10^6, f=2.15 cycles/sec
Observed: f=2.20 cycles/sec

Amplitude, A

At t = 0 A=1.13 in
t = 3.4 sec. A=0.33 in
(7.5 cycles)
t = 6.8 A=0.06 in
(15.0 cycles)

Logarithmic Decrement,

t_0 to $t_{3.4}$ δ =0.164
$t_{3.4}$ to $t_{6.8}$ δ =0.227
t_0 to $t_{6.8}$ δ =0.196

TABLE II -Ordinates of Decay Curves - Contra Costa Stacks - Unlined

	No. of	Amplitudes*					$\frac{A_o}{A_n}$	$\log_e$	δ
Sec.	Cycles	Run No. 1	Run No. 2	Run No. 3	Run No. 4	Avg.			
0.0	0.0	5.00	5.00	5.00	5.00	5.00			
2.5	2.43	4.56	4.58	4.68	4.81	4.66	1.07	.068	.028
5.0	4.85	4.13	4.16	4.25	4.56	4.28	1.09	.086	.036
7.5	7.28	3.76	3.75	3.77	4.22	3.88	1.10	.095	.039
10.0	9.71	3.38	3.33	3.26	3.77	3.43	1.13	.122	.050
12.5	12.14	3.00	2.94	2.84	3.37	3.04	1.13	.122	.050
15.0	14.56	2.68	2.65	2.51	3.00	2.71	1.12	.113	.047
20.0	19.42	2.14	2.12	1.99	2.42	2.17	1.25	.223	.046
25.0	24.27	1.84	1.74	1.66	2.00	1.81	1.20	.182	.038
30.0	29.13	1.61	1.48	1.40	1.68	1.54	1.17	.157	.032
35.0	33.98	1.35	1.29	1.14	1.40	1.30	1.18	.165	.034
40.0	38.83	1.13	1.08	0.99	1.18	1.10	1.18	.165	.034
45.0	43.69	0.88	0.83		0.99	0.90	1.22	.199	.041
Average							5.56	1.716	.039

* Amplitudes measured in inches

TABLE III Ordinates of Decay Curve - Contra Costa Stacks - Lined

No. of Sec.	Cycles	Run No. 2 Amp*	$\frac{A_o}{A_n}$	$\log_e$	δ	Run No. 3 Amp*	$\frac{A_o}{A_n}$	$\log_e$	δ	Run No. 4 Amp*	$\frac{A_o}{A_n}$	$\log_e$	δ	Run No. 5 Amp*	$\frac{A_o}{A_n}$	$\log_e$	δ	δ Ave.
0.0	0.0	5.00				5.00				5.00				7.33				
			1.22	0.199	0.112		1.20	0.182	0.103		1.22	0.199	0.112		1.43	0.358	0.202	0.132
2.5	1.77	4.11				4.17				4.11				5.11				
			1.23	.207	.116		1.22	.199	.112		1.19	.174	.098		1.17	.157	.088	.103
5.0	3.55	3.35				3.42				3.45				4.37				
			1.22	.199	.112		1.23	.207	.116		1.16	.148	.084		1.13	.122	.069	.095
7.5	5.32	2.74				2.79				2.97				3.86				
			1.22	.199	.112		1.18	.165	.093		1.13	.122	.069		1.12	.113	.064	.084
10.0	7.09	2.24				2.37				2.63				3.43				
			1.18	.165	.093		1.18	.165	.093		1.12	.113	.064		1.10	.095	.054	.076
12.5	8.87	1.89				2.01				2.34				3.10				
			1.19	.174	.098		1.10	.095	.054		1.08	.077	.043		1.13	.122	.064	.065
15.0	10.64	1.59				1.82				2.16				2.75				
			1.36	.307	.087		1.27	.239	.068		1.17	.157	.044		1.19	.174	.049	.062
20.0	14.18	1.17				1.43				1.84				2.31				
			1.39	.329	.093		1.22	.199	.056		1.16	.148	.042		1.20	.182	.051	.060
25.0	17.73	0.84				1.17				1.58				1.93				
			1.22	0.199	.056		1.32	.278	.078		1.19	.174	.049		1.18	.165	.046	.057
30.0	21.28	0.69				0.89				1.33				1.64				
							1.25	.223	.063		1.19	.174	.049		1.22	.199	.056	.056
35.0	24.82					0.71				1.12				1.34				
											1.08	.077	.022		1.21	.191	.054	.038
40.0	28.37									1.04				1.11				
Average			7.25	1.981	.093		7.08	1.952	.079		4.81	1.570	.055		6.60	1.887	.067	.074

* Amplitudes measured in inches

PROCEEDINGS-SEPARATES

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a. Presented at the New York (N.Y.) Convention of the Society in October, 1953.
c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.